Chapter 5

Fixed Income,
Zero coupon rates
and Duration

Fixed-income securities are tradable debt notes issued by an organization in which it declares that it will repay a borrowed amount with interest to the holder. Fixed-income securities can be issued in various ways: through an exchange or directly to the investor (over-the-counter). In general, fixed-income securities that are listed on an exchange can be easily traded, not only through the exchange but also through other public trading platforms or over-the-counter. The most well-known fixed-income securities that are traded through an exchange are bonds. Fixed-income securities that are directly placed with investors are usually part of a lending programme. This means that an organization can issue fixed-income securities up to a certain maximum amount with a relatively low administrative burden. Most issues through a programme are over-the-counter.

As with listed shares, fixed-income securities are registered on securities accounts with custodians or central securities depositories. This applies to all fixed-income securities, listed fixed-income securities like bonds and unlisted fixed-income securities like commercial paper or medium term notes.

5.1 Domestic, foreign & euro capital markets

Each currency has its own capital market in which transactions are concluded in the local currency. These capital markets are called local capital markets. When a party performs a transaction in a currency other than in the currency of the country where this party is resident then the word euro is added to this transaction. For example, when a British company in the UK issues a bond in USD, this is called a eurobond. Furthermore, if a U.S. company issues a bond in Japanese yen or a party issues a bond in its own currency but, however, not in its own country, these bonds are also called eurobonds. Finally, a bond in USD, issued by an American company in Europe, is also called a eurobond.
The following names are used for eurobonds issued by non-residents of the relevant currencies:

<table>
<thead>
<tr>
<th>MARKET</th>
<th>NICK NAME</th>
</tr>
</thead>
<tbody>
<tr>
<td>US</td>
<td>Yankee</td>
</tr>
<tr>
<td>GB</td>
<td>Bulldog</td>
</tr>
<tr>
<td>Japan</td>
<td>Samurai</td>
</tr>
<tr>
<td>Greece</td>
<td>Marathon</td>
</tr>
<tr>
<td>Spain</td>
<td>Matador</td>
</tr>
<tr>
<td>Canada</td>
<td>Maple</td>
</tr>
</tbody>
</table>

Eurobonds must satisfy the rules stipulated by the ICMA – the International Capital Market Association.

### 5.2 Issuing of fixed income securities

Fixed income securities can be issued in various ways. Some fixed income securities are listed on the exchange. The advantage of this is that generally makes them easier to trade. Other fixed income securities are placed with investors either directly or over-the-counter. The negotiability of these securities is generally low. Fixed income securities with an exchange listing can, however, also be traded over-the-counter.

#### 5.2.1 Pricing of fixed income securities

Before a bond is issued, the coupon rate is determined. This is generally done using a spread with respect to a benchmark such as government bonds or the yield on interest rate swaps.

For example, a bond is priced as ‘10 through swaps’. This means that the yield of the bond is 10 basis points below the swap rate for a comparable maturity period. For the auctioning of Dutch government bonds, for instance, the primary dealers subscribe by specifying a spread above a German ‘reference bond’.
5.2.2 **Exchange listed fixed income securities**

Some bonds have a stock exchange listing. The best known fixed income security with a listing is a bond. Bonds are negotiable certificates of indebtedness with a fixed interest coupon for the entire period to maturity. If an organization wants to list a fixed income security on an exchange then it needs, among other things, to make up a prospectus. This is a brochure specifying the business activities and economic data of the organization. An investor can use this information to form a judgement about the quality of the issuing organization.

For some bonds, there is an option to trade the coupons and the principal separately. Each individual coupon and the principal sum is then considered as a separate security and is given its own ISIN code (International Securities Identification Number). This phenomenon is called STRIPS (also known as coupon stripping). STRIPS stands for Separate Trading of Registered Interest and Principal Securities.

For most exchange issues, the fixed income securities are issued in one go. However, some fixed income securities are issued at more than one moment. This is the case, for example, for government bonds and these are known as seasoned issues.

When government bonds are issued, there are only a limited number of banks that may disperse them amongst the investors. For Dutch government bonds, these banks are called the primary dealers and, for UK government bonds, they are called Gilt Edged Market Makers GEMM. Only the primary dealers may participate in an auction. Other expressions for an auction include ‘tap’ or ‘tender’. In each country there is a separate government organization that issues these bonds and organizes the auctions. In the US, for instance, this is the US Treasury. After the primary dealers have subscribed, this body determines how many bonds it wants to issue and what the yield is.

All subscriptions made at this or lower yields are allocated. Sometimes all subscriptions are allocated against the same fixed yield. Thus, there is no so-called ‘winner’s curse’. This method is called a non-competitive auction. In other cases, the allocating body uses the yield that the banks have individually subscribed. That is the case with, for instance, Belgian, Portuguese and Irish government bonds.

Government bonds are often traded at dedicated trading systems. British government bonds are, for instance, traded via the Inter Dealer Brokers system (IDB). Dutch government bonds are traded via four systems: ICAP Electronic Broking, Eurex Bonds, BGC Brokers LP and MTS.
5.2.3 Private placement (over-the-counter)

Applying for a listing and producing a brochure is time consuming and costly. For this reason, many organizations choose to issue fixed income securities privately (over-the-counter). This means that they conclude a contract directly with the lender, without the intervention of a public marketplace. In this case, however, a bank will also generally play the role of intermediary.

A private loan is a loan agreement between a borrower and one or more lenders, often institutional investors. In addition to the fact that the issue costs for a private loan are much lower for the borrower, the administration and logistics involved in this method of borrowing are also simpler. The entire sum loaned is fixed in one or more formal contracts and the interest and principal are generally transferred directly to the lender. Furthermore, the lender does not need to pay any custodian fees, since there are no securities involved that have to be registered by a custodian or central securities depository.

The interest rate in the private market (over-the-counter) is generally somewhat higher than that in the public market (exchange). This is because of the low marketability. This is because an investor in private loans has large, unmarketable, items in his portfolio and wants to be compensated for this.

Many loans concluded outside the exchange fall under a loan programme. Such a programme means that a borrower can issue interest-bearing securities up to a specific maximum amount without having to request an exchange listing and without the need to produce a separate prospectus each time. Thus, with a loan programme there is a recurring issue, initiated by the borrower. The utilization of a loan programme is called drawing. The drawing on a programme can be either on the initiative of the issuer or on the initiative of the investor. Within a loan programme, various kinds of fixed income securities can be issued. Examples include medium term notes and commercial paper. In addition, bonds can also be issued under a loan programme. For these bonds, a separate prospectus must be issued but this is not required to be as extensive as that for a bond not issued under a loan programme. A lending programme is also called a debt issuance programme (DIP) or note issuance facility (NIF).

Settlement

Bonds are settled at central securities depositories such as Euroclear, Clearstream and Crestco in Europe and the DTC in the US. Eurobonds are settled after three working days and UK Gilts, after one working day. Generally speaking, settlement takes place on a delivery versus payment basis (DVP). This means that the transfer of the securities and money take place simultaneously in the settlement system of the settlement institution. The selling party must ensure that the securities are in
its securities account at the settlement institution and the buying party must ensure that the money is in its cash account at the settlement institution. If the settlement institution determines that both parties have complied with their obligation, it will transfer the securities and the money simultaneously.

For the settlement of UK Gilts at Crestco, the settlement is not DVP – but the principle of assured payment is used. This means that the buying party does not have to transfer the money to Crestco but that Crestco guarantees to the selling party that the money will be received. If a buyer then fails to transfer the money to Crestco, Crestco will take on the payment obligation itself and recover the money later from the buying party.

5.3 The yield of a fixed-income security

The yield of a fixed-income security is the return that investors demand, based on the assumption that they will keep the bond until the end of the term. This is why it is also called yield to maturity. Investors consider the following factors when determining the yield they request on a fixed-income security:

- the general interest rate level on the capital market;
- the quality of the issuing entity;
- the liquidity of the bond.

5.3.1 The general interest rate level on the capital market

The capital market interest rate is determined by the free interplay between supply and demand. The interest rate level on the capital market is largely determined by the expected inflation. When investors expect a higher rate of inflation, they demand higher compensation (‘inflation compensation’). This is because inflation lowers the value of the amortization in real terms. A higher (expected) inflation is therefore coupled with an increase in the capital market interest rate. Conversely, a lower expected inflation leads to a decrease in the capital market interest rate.

In addition to the inflation expectation, supply and demand factors also play a role. The capital market interest rate may fall due to an increase in the demand for fixed-income securities. This may be the case, for instance, when investors liquidate their share investments en masse, so that more money becomes available for fixed-income securities.

A well-known benchmark on the capital market is the return on government loans. Nowadays, however, this benchmark is no longer commonly used. Instead, the in-
The interest rate swap (IRS) rate level is now often used as a benchmark. These are inter-
bank rates, just like the Euribor rates. The benchmark that applies to IRS rates is the
ISDAFIX Euribor Swap Rate, which is fixed at 11.00 AM every day as an average of
the quotes given by several dealers selected by ISDA, ICAP and Thomson Reuters.
The ISDAFIX Euribor swap rates are fixed for the following periods: for 1-10 years
and for 12, 15, 20, 25 and 30 years.

### 5.3.2 The quality of the issuing entity

The quality or creditworthiness of the issuing organization indicates the likelihood
of it being able to pay back the loan. The better the quality, the more confident the
investor is that he will get his money back. An investor who buys a bond issued by
a central government is usually almost certain that he will get his money back. Figure 5.1 below shows some examples of government bonds and their characteristics.

<table>
<thead>
<tr>
<th>COUNTRY</th>
<th>NAME</th>
<th>TERM</th>
<th>COUPON FREQUENCY</th>
</tr>
</thead>
<tbody>
<tr>
<td>Germany</td>
<td>Bund</td>
<td>10 yr</td>
<td>annual</td>
</tr>
<tr>
<td>Germany</td>
<td>Bobl</td>
<td>5 yr</td>
<td>annual</td>
</tr>
<tr>
<td>Germany</td>
<td>Schatz</td>
<td>2 yr</td>
<td>annual</td>
</tr>
<tr>
<td>Japan</td>
<td>JGB</td>
<td>10 yr</td>
<td>semi-annual</td>
</tr>
<tr>
<td>Japan</td>
<td>Super Long Bond</td>
<td>20 yr</td>
<td>semi-annual</td>
</tr>
<tr>
<td>Italy</td>
<td>BTP</td>
<td>3-30 yr</td>
<td>semi-annual</td>
</tr>
<tr>
<td>Spain</td>
<td>Bono</td>
<td>3-30 yr</td>
<td>annual</td>
</tr>
<tr>
<td>France</td>
<td>OAT</td>
<td>3-50 yr</td>
<td>annual</td>
</tr>
<tr>
<td>US</td>
<td>Treasury Bond</td>
<td>10-30 yr</td>
<td>semi-annual</td>
</tr>
<tr>
<td>US</td>
<td>Treasury Note</td>
<td>2-10 yr</td>
<td>semi-annual</td>
</tr>
<tr>
<td>UK</td>
<td>Gilt</td>
<td>10 yr</td>
<td>semi-annual</td>
</tr>
</tbody>
</table>

Companies are usually less creditworthy than governments. In order to compensate
for this, investors demand higher coupon rates for securities issued by companies
compared to government bonds. The difference in interest rate between the cou-
pon that a certain institution must pay and that which the government must pay is
called the risk premium or credit spread.

In order to determine the creditworthiness of issuing institutions, many investors
use ratings. A rating is an assessment by a so-called rating agency of an organiza-
tion’s creditworthiness, expressed as a letter or a combination of letters and digits. Rating agencies make a rough distinction between two categories of issuers: investment grade and high yielders. If a bond is qualified as investment grade, it means that it is a reasonably safe investment. The categories of ratings used by Moody’s and Standard & Poor’s are shown in figure 5.2.

![Figure 5.2 Overview of credit ratings](image)

<table>
<thead>
<tr>
<th>INVESTMENT GRADE</th>
<th>MOODY’S</th>
<th>S&amp;P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Highest quality</td>
<td>Aaa</td>
<td>AAA</td>
</tr>
<tr>
<td>High quality</td>
<td>Aa</td>
<td>AA</td>
</tr>
<tr>
<td>Upper medium grade</td>
<td>A</td>
<td>A</td>
</tr>
<tr>
<td>Adequate payment capacity</td>
<td>Baa3</td>
<td>BBB-</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>HIGH YIELDERS</th>
<th>MOODY’S</th>
<th>S&amp;P</th>
</tr>
</thead>
<tbody>
<tr>
<td>Predominantly speculative</td>
<td>Ba</td>
<td>BB+</td>
</tr>
<tr>
<td>Speculative, low grade</td>
<td>B</td>
<td>B</td>
</tr>
<tr>
<td>Poor quality/Vulnerable to default</td>
<td>Caa</td>
<td>CCC</td>
</tr>
<tr>
<td>Highest speculation</td>
<td>Ca</td>
<td>CC</td>
</tr>
<tr>
<td>Lowest quality, no interest being paid</td>
<td>C</td>
<td>C</td>
</tr>
<tr>
<td>Questionable value</td>
<td>C</td>
<td>DDD</td>
</tr>
<tr>
<td>In arrears</td>
<td>D</td>
<td>DD</td>
</tr>
<tr>
<td>In bankruptcy</td>
<td>D</td>
<td>D</td>
</tr>
</tbody>
</table>

In addition to the quality of the issuing organization itself, the domicile also plays a role with regard to the creditworthiness of a client. Governments sometimes impose restrictions on the payment of interest and amortization of foreign debts. This is called political risk. The political risk is included in the rating of individual parties. If a political risk exists, an investor will of course demand a higher yield.

### 5.3.3 The liquidity of the bond

Liquidity with respect to financial instruments means tradability. If bonds are not regularly traded, they are less appealing to investors and they will demand a higher yield. This is because investors need to wait and see if there is a counterparty that is interested and how much this party is willing to pay should they want to sell these
bonds. The liquidity of a bond is closely related to the number of bonds issued. Generally speaking, the liquidity increases as more bonds of a certain type are issued. In order to improve the liquidity, the Dutch State Treasury Agency, for instance, only issues one new bond a year in principle. When the Dutch State needs money again, the DSTA reopens this bond, leading to an increase in the bond size and thus also in liquidity.

5.4 Special types of fixed-income securities

A wide range of fixed-income securities are issued in the financial markets, each with its own features.

FLOATING RATE NOTES

A floating rate note (FRN) is a fixed-income security with an interest coupon that is reset periodically, usually after three or six months. The reference rate of an FRN is usually a money market benchmark such as LIBOR or EURIBOR.

ASSET-BACKED SECURITIES AND COVERED BONDS

Asset-backed securities are issued as a result of a securitization transaction. With securitization, the initiating party transfers a portion of its assets to a third party. This is often a legally independent entity created specifically for this purpose, known as a special purpose vehicle (SPV). The SPV obtains its funds by issuing debt securities in the form of bonds, medium-term notes or commercial papers.

Securitization is not the actual sale of assets. If this was the case and a bank wanted to securitize a loan portfolio, all contracts between the bank and the borrowers would have to be replaced by contracts between the SPV and the borrowers. In order to prevent this, credits are only transferred in an economic sense and not in a legal sense. This means that the bank has to arrange for all cash flows arising from the transferred portfolio to be forwarded to the SPV. The SPV can in turn use these cash flows to pay interest and instalments for the fixed-interest securities it has issued.

The cash flows of the acquired receivables thereby serve as a kind of collateral for payment obligations resulting from securities issued by the SPV. These interest-bearing securities are therefore called asset-backed securities (ABS) or collateralized debt obligations (CDOs). Apart from the duty to channel all cash flows of the securitized portfolio to the SPV, the bank has no further obligations towards the SPV.
A variation on asset-backed securities are covered bonds. These are bonds issued by a bank which are collateralized by a specific credit portfolio.

CALLABLE/PUTABLE BONDS

Callable notes are fixed-income securities that can be redeemed prematurely by the issuer. The issuer only makes use of this option if the current interest rate is lower than the coupon rate. This is not a favourable option for investors as they have to invest their funds at a lower rate. Loan terms and conditions, therefore, often stipulate that callable bonds are redeemed at a price at or above 100 in the event of premature redemption.

A putable note or bond is a fixed-income security that the holder is entitled to sell prematurely to the issuer under predetermined conditions.

PERPETUALS

Perpetuals are fixed-income securities that do not need to be redeemed. Interest on most perpetuals, however, is periodically reviewed, e.g. every 10 years, which means that the coupon is adjusted to prevalent market rates every 10 years. Investors are thereby given the assurance that interest yields will not vary from prevalent market rates for too long. Most perpetual bonds are callable and they can be redeemed prematurely at intervals as defined in the loan agreement.

SUBORDINATED LOANS OR JUNIOR LOANS

Subordinated loans or junior loans are loans in which interest payments and repayments are subordinated to other fixed-income securities of the issuer. In the event of payment difficulties, the issuer must first settle obligations in respect of other loans before it is required to comply with interest payments and/or to repay a junior loan. Investors thereby run a greater risk of not getting their money back than with a regular fixed-income security. In return for this risk, they receive a higher interest rate.

INFLATION-_LINKED BONDS

Inflation-linked bonds or index-linked bonds are fixed-income securities in which the nominal value is periodically adjusted for inflation. Interest is also calculated against the higher amount. In case of inflation, the investor sometimes also receives a higher amount at settlement than the initial investment.
EXAMPLE

A pension fund invests in a one-year index-linked bond with a nominal value of EUR 1,000.00 and a coupon yield of 1%, with the guarantee that the principal amount is indexed to inflation.

With an inflation rate of 2.4%, the pension fund will be repaid the principal amount of EUR 1,024.00, plus 1% interest after one year (1% of EUR 1,024.00 = EUR 10.24). The total amount payable after one year is EUR 1,034.24.

Figure 5.3 shows examples of inflation-linked government bonds.

**Figure 5.3** Inflation-like bonds

<table>
<thead>
<tr>
<th>LAND</th>
<th>BOND NAME</th>
<th>UNDERLYING INFLATION INDEX</th>
</tr>
</thead>
<tbody>
<tr>
<td>Great Britain</td>
<td>Inflation Linked Gilt (ILG)</td>
<td>Retail Price Index (RPI)</td>
</tr>
<tr>
<td>United States</td>
<td>I-Bond</td>
<td>US Consumer Price Index</td>
</tr>
<tr>
<td>France</td>
<td>OATi</td>
<td>France CPI</td>
</tr>
<tr>
<td>Germany</td>
<td>Bund index and BOBL index</td>
<td>EU HICP</td>
</tr>
<tr>
<td>Japan</td>
<td>JGBi</td>
<td>Japan CPI</td>
</tr>
<tr>
<td>Italy</td>
<td>BTPi</td>
<td>EU HICP</td>
</tr>
</tbody>
</table>

JUNK BONDS

Junk bonds or high-yielders are fixed-income securities issued by companies with low credit ratings. Investors buy these fixed-income securities because they offer a high return. A fixed-income security can also be downgraded to junk bond status during its lifetime. This happens when the creditworthiness of the issuer falls sharply during its lifetime. The fixed-income security is then referred to as a fallen angel. High-yield bonds or junk bonds can be identified by a credit rating lower than Baa3 or BBB-. 
CONVERTIBLE BONDS

Convertible bonds are bonds that can be exchanged by the holder during their lifetime for a pre-determined number of shares in the issuer. The holder of a convertible bond actually holds a combination of a bond and a call option on the issuer’s shares. The option premium is included in the interest rate, and this rate is, therefore, lower than the interest rate on a regular bond with the same maturity.

ZERO COUPON BONDS

Zero coupon bonds are fixed-income securities that do not yield any interim coupons during their lifetime. Zero coupon bonds are redeemed at their face value. The issuing price is calculated as the present value of the face value. Zero coupon bonds are popular among investors because they run no reinvestment risk, which is also why they are often used as a hedge for guarantee products.

5.5 The price of a bond

For a transaction in a fixed-income security, the accrued interest until the settlement date is also purchased. Thus, the price of a bond is made up of the market price plus the accrued coupon interest up until the settlement date. The price without the accrued interest is called the clean price. The price including the accrued interest is called the dirty price.

EXAMPLE

An asset manager buys a fixed income security with a nominal value of USD 10,000,000 at a clean price of USD 9,860,000. The annual interest coupon is 5% (actual/actual) which is paid out on 15 July.

If the settlement date for this purchase is 10 July, the dirty price of the bond is USD 9,860,000 + USD 10,000,000 x 360/365 x 5% = USD 9,860,000 + USD 493,150.68 = USD 10,353,150.68.

If the settlement date for this purchase is 20 July, the dirty price of the bond is USD 9,860,000 + USD 10,000,000 x 5/365 x 5% = USD 9,860,000 + USD 6,849.31 = USD 9,866,849.31.
Before buying a fixed income security, investors first calculate the price that they would be willing to pay for it. This price is based on the yield they want to realize. They do so by calculating the present value of all the individual cash flows of the security, and then adding together these present values. Here, the yield is used as the discount rate. The result is the dirty price of the bond. The clean price the investor wants to pay is determined by subtracting the accrued interest from the dirty price.

The equation used to calculate the present value of each of the future cash flows of a bond that matures after more than 1 year with annual coupon payments is:

\[
\text{Present value} = \frac{\text{Cash flow}}{(1 + \text{interest rate})^{\text{period}}}
\]

For broken periods, the exponent is expressed as a number of whole years and a fraction of the year in which the broken period falls.

How the exponent of the number of years is determined depends on the applicable day count fraction. If the government bond yield curve is used for discounting, the day count fraction is actual/actual. For a broken period, the exact number of days in the period must be calculated in the broken year and then this number is divided by the total number of days in the entire year (365 or 366). If the interest rate swap yield curve is used for discounting, the day count fraction is 30/360. With a broken period, the number of days in the broken period is now calculated by setting each month at 30 days.

The price of a listed bond is always a clean price. It is expressed as a percentage of the principal where the %-sign is left out. The price of a listed bond is, therefore, for instance 99.8 or 104.17. The clean price of a bond with annual coupons can be calculated by using the following equation:\(^{35}\).

\[
\text{price} = 100 \times \left[ \frac{C}{r} \times \left( 1 - \frac{1}{(1+r)^n} \right) \right] + \frac{1}{(1+r)^n}
\]

where
\[n = \text{remaining term of the bond} ; \]
\[c = \text{coupon} \% ; \]
\[r = \text{yield} \ (4.00\% = 0.04). \]

\(^{35}\) The equation to calculate the clean price of a bond should be entered in a HP Financial Calculator as follows: \(BOND = 100 \times \left( \frac{C\%}{Y\%} \times \frac{1-1}{(1+Y\%)^N} \right) + \frac{1}{(1+Y\%)^N}. \)
The above method is also used to calculate the clean price of unlisted fixed income securities.

Figure 5.4 shows the calculation of the clean price of a BASF bond (EUR 1,000,650.00) and its accrued interest (EUR 36,712.33).

Figure 5.4  Thomson Reuters calculation tool for bonds
5.6 Zero coupon rates

As we have seen, the yield can be used to determine the present value of a bond. However, to determine the present value of individual cash flows such as the principal of a zero coupon bond, another kind of interest rate is used. This rate is called the zero coupon rate.

The difference between yield and zero coupon rate can be explained with an example of an investment of EUR 1,000 in a fixed income security where an investor has the choice between, on the one hand, a newly issued ordinary bond with a maturity period of two years and a coupon of 4% and, on the other hand, a zero coupon bond with a maturity period of also two years, both issued by the same issuer.

At the issue date, the yield curve is as follows:

<table>
<thead>
<tr>
<th>PERIOD</th>
<th>YIELD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>3.90</td>
</tr>
<tr>
<td>2 years</td>
<td>4.00</td>
</tr>
</tbody>
</table>

To calculate the yield for the zero coupon bond that will provide the same investment result for the investor as the ordinary bond, the maturity value of the investment in the ordinary bond after two years must first be calculated. This maturity value is made up of three parts:

- the principal: EUR 1,000
- the coupons 2 x EUR 40
- the reinvestment result for the first coupon.

Theoretically, the investor should be able to invest the first coupon against the forward rate for the forward period of one year starting after one year. This forward rate is 4.10410 and the amount of interest that the investor receives from the first coupon would be 0.0410410 x EUR 40 = EUR 1.64164164164.

The total maturity value of the investment in the regular bond after two years is therefore EUR 1,081,641.64. This must also be the maturity value for the zero coupon bond, since the two investment alternatives are identical with regard to time and risk.

If the maturity value of the zero coupon bond is EUR 1,081,641.64, the present value equation can be used to calculate the yield on this coupon: this is the two-year ‘zero coupon rate’.
The difference between the zero coupon rate and the normal yield can be explained by examining the reinvestment results of the coupons more closely. In a rising yield curve, the coupons can be reinvested against higher forward rates than the yield of the bond. In the above example, for a yield of 4.0%, the expected maturity value, using the yield as the reinvestment rate, is only EUR 1.000 \times (1 + 0.04)^2 = EUR 1,081.60. Because the expected future value is in reality higher, i.e. EUR 1,081.64, the zero coupon rate must be higher than the yield of the bond. For an inverse yield curve, the opposite reasoning applies.

Zero coupon rates can be calculated for every regular yield curve. The method used for this is called bootstrapping. Bootstrapping makes use of two principles:

1. The zero coupon rate only differs from the yield in the case of compounded interest - thus generally for periods of longer than a year

2. The price of a bond with a coupon that is equal to the yield for the remaining period of this bond (a par bond) is 100.

On the basis of the first principle, the one year zero coupon rate is the same as the one year yield. To calculate the two year zero coupon rate, we use the second principle and apply it to a two-year par bond. The yield curve is as follows

<table>
<thead>
<tr>
<th>PERIOD</th>
<th>YIELD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>2.29</td>
</tr>
<tr>
<td>2 years</td>
<td>2.59</td>
</tr>
<tr>
<td>3 years</td>
<td>2.78</td>
</tr>
</tbody>
</table>

A two-year par bond with a nominal value of EUR 1,000 therefore has a coupon of 2.59% and a price of 100. This price is calculated as the sum of the present values of all cash flows arising from the bond:

\[
\text{Price} = \frac{25.90}{1 + 0.0229} + \frac{1025.90}{(1 + 2 \text{ yr z.c. yield})^2} = EUR 1,000
\]
The present value of the first cash flow of EUR 25.90 is EUR 25.32. The present value of the second cash flow is then equal to EUR 1,000 \(/\) EUR 25.32 = EUR 974.68. The following equation can be used to determine the two year zero coupon rate (z.c._2):

\[
\text{Present value} = \frac{1025.90}{(1 + 2 \text{ yr z.c. yield})^2} = 974.68
\]

and then

\[
z.c._2 = \left[\sqrt[2]{\frac{1025.9}{974.68}} - 1\right] \times 100\% = 2.5939\%
\]

A three year par bond can be used to determine the three year zero coupon rate. The cash flows for a three year par bond with a nominal value of EUR 1,000 are respectively EUR 27.80, EUR 27.80 and EUR 1027.80. The present value of these cash flows are respectively:

<table>
<thead>
<tr>
<th>TERM</th>
<th>CASH FLOW</th>
<th>DISCOUNT FACTOR</th>
<th>PRESENT VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>27.8</td>
<td>1/1.0229</td>
<td>27.18</td>
</tr>
<tr>
<td>2 years</td>
<td>27.8</td>
<td>1/1.025939</td>
<td>26.41</td>
</tr>
<tr>
<td>3 years</td>
<td>1027.8</td>
<td>to be calculated</td>
<td>946.41</td>
</tr>
</tbody>
</table>

The present value of the first cash flow of EUR 27.80 is EUR 27.18 (calculated with the one year yield) and the present value of the second cash flow of EUR 27.80 is EUR 26.41 (calculated with the two year zero coupon rate that we just have found). Since the price and thus the sum of the present values is equal to EUR 1,000, the present value of the third cash flow must be EUR 1,000 \(/\) EUR 27.18 \(/\) EUR 26.41 = EUR 946.41. The following equation can now be used to determine the three year zero coupon rate (z.c._3):

\[
\text{Present value} = \frac{1027.8}{(1 + 3 \text{ yr z.c. yield})^3} = 946.41
\]

and then

\[
z.c._3 = \left[\sqrt[3]{\frac{1027.8}{946.41}} - 1\right] \times 100\% = 2.7881\%
\]
This method can be repeated for longer periods. Use is always made of a par bond with a price of EUR 1,000. Next, with the exception of the last cash flow, the present value of all cash flows is calculated using the one year yield and the already calculated zero coupon rates. In this way, there is sufficient information available to calculate the present value of the last cash flow. The zero coupon rate is then always calculated by using the following equation\(^{36}\):

\[
ZC_{n} = \left[ \frac{\text{Future value last cash flow}}{\sqrt[n]{\text{Present value last cash flow}}} - 1 \right] \times 100\%
\]

The table below shows regular yields, also called coupon yields, and the corresponding zero coupon rates for periods from one to ten years.

<table>
<thead>
<tr>
<th>PERIOD</th>
<th>(COUPON) YIELD</th>
<th>ZERO-COUPON YIELD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2.29</td>
<td>2.2900</td>
</tr>
<tr>
<td>2</td>
<td>2.59</td>
<td>2.5939</td>
</tr>
<tr>
<td>3</td>
<td>2.78</td>
<td>2.7881</td>
</tr>
<tr>
<td>4</td>
<td>2.92</td>
<td>2.9327</td>
</tr>
<tr>
<td>5</td>
<td>3.05</td>
<td>3.0689</td>
</tr>
<tr>
<td>6</td>
<td>3.15</td>
<td>3.1746</td>
</tr>
<tr>
<td>7</td>
<td>3.23</td>
<td>3.2600</td>
</tr>
<tr>
<td>8</td>
<td>3.29</td>
<td>3.3242</td>
</tr>
<tr>
<td>9</td>
<td>3.34</td>
<td>3.3783</td>
</tr>
<tr>
<td>10</td>
<td>3.36</td>
<td>3.3981</td>
</tr>
</tbody>
</table>

For bootstrapping, it is not necessary to use a regular yield curve. A zero coupon curve can also be derived from the prices of a number of random bonds with different maturity periods.

EXAMPLE

Given are the one-year rate of 4% and the price of a 5% bond with a remaining maturity period of two years of 101.70.

\[ ZC2\% + 1 = \text{SQRT}\left(\frac{\text{P+C}}{\text{P-C} / (1+ZC1\%)}\right) \]
The two year zero coupon yield can now be calculated using the following equation:

\[
z_{c.2} = \left[ \frac{105}{\sqrt[4]{101.70 - 5/1.04}} - 1 \right] \times 100\%
\]

The two year zero coupon rate is: 4.94\%\(^{37}\).

5.7 Relationship between bond price and yield, modified duration

The value of interest rate instruments such as fixed income securities and interest rate derivatives changes when the interest rate changes. This is because this value is calculated as the sum of the present values of the cash flows of the fixed income security. If the (coupon) yields change, the zero coupon rates also change and, therefore, also the present value of the constituent cash flows. As a consequence, the market value of the interest rate instrument changes. This can be demonstrated using the present value equation.

\[
\text{Present value} = \frac{\text{Cash flow}}{(1 + \text{interest rate})^\text{period}}
\]

If there is a rise in the interest rate, the value of the denominator of the fraction increases and, therefore, the present value decreases. For a decrease, the reverse is true. This means that the value of an interest rate instrument falls with an interest rate rise and increases with an interest rate fall.

The modified duration provides an indication of the interest rate sensitivity of a particular interest rate instrument or a portfolio of interest rate instruments. Modified duration is an elasticity that sets the percentage change in the dirty price of an instrument due to a change in interest rate against this interest rate change. The equation to calculate the modified duration is:

\[
\text{Modified duration} = \frac{\% \text{ change of dirty price}}{\text{interest rate change}}
\]

\(^{37}\) Use the ZC2% equation in your HP Financial Calculator to calculate the two year zero coupon rate: \(\text{COUP} = 5, \text{PRICE} = 101.70, ZC1\% = 0.04\). Solve for ZC2\%.
EXAMPLE

The price of a bond is 98.45 and the duration of the bond is 4.72. This means that for a rate fall of 1 basis point (= 0.01%), the price of the bond will rise by 0.0472 x 98.45% = 0.0465 to 98.4965.

Using the modified duration, an estimate can be made of the risk associated with a position in a portfolio of fixed income securities. If a portfolio has a high modified duration, this means that the market value of this portfolio reacts strongly to interest rate changes. Thus, the market risk of this portfolio is high. The modified duration can also be used to provide an indication of the size of the hedge transactions required to cover the market risk of interest rate positions.

5.7.1 Calculation of the modified duration

The equation for calculating the modified duration of fixed income securities is as follows

\[
\text{Modified duration} = \frac{1}{1 + r} \times \frac{\sum_{i=1}^{n} i \times \text{present value coupon}_i + n \times \text{present value nominal}}{\sum_{i=1}^{n} \text{present value coupon}_i + \text{present value nominal}}
\]

where

\( n \) = remaining term for the fixed income security;
\( i \) = period of a cash flow;
\( r \) = yield (4.00% = 0.04).

Firstly, the present values of all the individual cash flows are calculated (coupons and principal). These present values are then used as a weighting factor to determine an average duration for the fixed income security. The average duration thus determined (called the Macaulay duration or just duration), is represented by the part of the above equation at the extreme right.

To determine the interest rate sensitivity, an ‘adjustment factor’ also needs to be applied. This is the factor ‘\( 1 / (1 + r) \)’ in the first part of the right side of the equation. The modified duration derives its name from this ‘adjustment factor’. The ‘\( r \)’ in the equation represents the effective yield of the financial instrument for which the modified duration is calculated. If the modified duration is known, the Macaulay duration can easily be calculated and vice versa.
The modified duration of a bond with a remaining maturity period of 2.5 years (day-count convention 30/360) and an interest coupon of 6% is calculated. Furthermore, it is known that all zero coupon rates are 4%.

Firstly, the present values of the three cash flows for this bond are calculated (column 3) and then each present value is multiplied separately by the corresponding period (column 4):

<table>
<thead>
<tr>
<th>TERM</th>
<th>CASH FLOW</th>
<th>PV CASH FLOW</th>
<th>PV X TERM</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 year</td>
<td>60</td>
<td>60/(1.04) = 57.69</td>
<td>57.69</td>
</tr>
<tr>
<td>2 years</td>
<td>60</td>
<td>60/(1.04)^2 = 55.47</td>
<td>110.94</td>
</tr>
<tr>
<td>3 years</td>
<td>60</td>
<td>1060/(1.04)^3 = 942.34</td>
<td>2,827.01</td>
</tr>
<tr>
<td>Σ</td>
<td>1,055.50</td>
<td>2,995.65</td>
<td></td>
</tr>
</tbody>
</table>

The modified duration of this bond is

\[
\text{Modified duration} = \frac{1}{1 + 0.04} \times \frac{2995.65}{1055.50} = 2.73
\]

The modified duration of a fixed income instrument can also be calculated with another equation. Unlike the previous equation, this equation is, however, suitable for programming in a calculator:

\[
\text{Modified duration} = \frac{\text{COUP}}{r^2} \times \left[ 1 - \frac{1}{(1 + r)^n} \right] + \frac{n \times (100 - \text{COUP})}{[1 + r]^{n + 1}}
\]

where

- \text{COUP} = \text{coupon payment};
- \text{Y\%} = \text{yield (4\% = 0.04)};
- n = \text{remaining term}.

38 The equation to calculate the modified duration should be entered in a HP Financial Calculator as follows:

\[
\text{MD} = (\text{C\%} \times 100 / \text{Y\%}^{\times 2} \times (1-1/(1+\text{Y\%})^N) + N \times (100-\text{C\%} \times 100 / \text{Y\%}) / (1+\text{Y\%})^{(N+1)}) / (100 \times ((\text{C\%} / \text{Y\%} \times (1 - (1+\text{Y\%})^N)) + 1 / (1+\text{Y\%})^N).
\]
5.7.2  Factors that determine the level of the modified duration

In general, the modified duration of an interest rate instrument increases as the remaining term of the instrument increases. The modified duration is after all largely determined by the Macaulay duration and this provides the average remaining term of the cash flows weighted by the size of the present value of the cash flows. An instrument with a longer remaining term therefore has a higher modified duration than an instrument with a shorter remaining term.

Because the remaining terms for the cash flows are weighted with to the size of the cash flows then, if the remaining term is the same, a bond with a higher coupon has a lower duration than a bond with a lower coupon. This is because for a bond with a higher coupon rate, the shorter periods will weigh more heavily.

Furthermore, the duration of a bond depends on the level of the market interest rate. For a high market interest rate, the duration is lower than for low interest rates. For higher interest rates, the present value of the principal falls more strongly than the present value of the coupons. Thus, for higher interest rates, the remaining term of the principal weighs less heavily than for lower rates and the duration of a bond (average period) is therefore lower.

5.7.3  Convexity

The modified duration suggests a linear relationship between the price and the interest rate. In fact, however, this relationship is not linear for a random bond but convex as shown in figure 5.5.
The following is known for a particular bond:

<table>
<thead>
<tr>
<th>COUPON</th>
<th>6%</th>
</tr>
</thead>
<tbody>
<tr>
<td>remaining term</td>
<td>3 year</td>
</tr>
<tr>
<td>price</td>
<td>105.55</td>
</tr>
<tr>
<td>duration</td>
<td>2.73</td>
</tr>
<tr>
<td>face value</td>
<td>1,000.00</td>
</tr>
</tbody>
</table>

Using the modified duration for an interest rate rise of 20 basis points, for instance, from 4.00% to 4.20%, the price decreases according to the equation:

\[
\text{Modified duration} = \frac{\% \text{ change of price}}{\text{interest rate change}}
\]

Therefore, with 2.73% x 0.20% = 5.46%.

In reality however, figure 5.1 shows that the price only decreased from 1055.50 to 1049.76. If this decrease is expressed as a percentage, we get

\[
\% \text{ change of price} = \frac{1049.76 - 1055.50}{1055.50} \times 100\% = 0.544\%
\]

In the above example, the actual decrease in the price is smaller than might be expected with the linear relationship that the modified duration suggests. With an interest rate fall, the actual price increase is however greater than might be expected based on the modified duration. The linear relationship between interest rate changes and price changes can only be assumed for very small changes in interest rate. Since the actual change in value is more favourable than expected, the term positive convexity is used in this respect.

### 5.7.4 Adding and subtracting the modified duration of different portfolios

With the help of the modified duration, it is possible to make quick comparisons of the risks associated with two or more different portfolios, to add the risks associated with portfolios and to determine the size of hedge transactions required to achieve a desired reduction in the risk. As an aid with these calculations, there is a variable that is sometimes called the money duration. The money duration of a
The modified duration of a fixed income security or portfolio is the product of the market value and the modified duration:

\[
\text{Money duration} = \text{dirty price} \times \text{modified duration}
\]

The following shows how the modified duration of a mixed portfolio can easily be calculated by using this concept.

\[\text{EXAMPLE}\]

A bond trader has a bond portfolio with a market value of EUR 180 million and a modified duration of 10.
The bond trader is considering whether to invest in another bond portfolio with a market value of EUR 100 million and a modified duration of 7.2.

In order to get an indication of the interest rate sensitivity of the composed portfolio the bond trader calculates its modified duration. To do this, he uses the ‘money duration’.

\[
\begin{align*}
\text{Money duration old portfolio:} & \quad \text{EUR} \, 180 \text{ million} \times 10 = (\text{EUR}) \, 1800 \text{ million.} \\
\text{Money duration new portfolio:} & \quad \text{EUR} \, 100 \text{ million} \times 7.2 = (\text{EUR}) \, 720 \text{ million.} \\
\text{Money duration composed portfolio (} = \text{sum of money durations):} & \quad (\text{EUR}) \, 2520 \text{ million.} \\
\end{align*}
\]

Using the money duration equation, the modified duration of the composed portfolio can also easily be calculated:

\[
\text{EUR} \, 2520 = \text{EUR} \, 280 \text{ million} \times \text{modified duration.}
\]

From this it follows that the modified duration of the composed portfolio is EUR 2,520 million / EUR 280 million = 9.

The ability to add and subtract the risks for two interest bearing portfolios is not only just applicable for portfolios consisting of identical instruments but for all interest bearing portfolios. Thus, for example, the risks for a bond portfolio, a portfolio of purchased bond futures and a portfolio of receiver’s interest rate swaps can be added. On the other hand, the risks for a bond portfolio and for a portfolio of sold bond futures used as a hedge can be subtracted from each other.

Using the money duration, a portfolio manager can easily determine the size of a hedge transaction that he must conclude if he wants to adjust the modified duration.
of his portfolio. The conclusion of a contract in other financial instruments with the aim of reducing the duration of an existing portfolio is referred to as duration hedge.

EXAMPLE

A bond trader has a bond portfolio with a market value of EUR 180 million and a modified duration of 10. He fears a rise in interest rates and therefore wants to decrease the modified duration of this portfolio to 5. To achieve this, he plans to sell bond futures with a modified duration of 9.

The money duration of the bond portfolio must thus be reduced from EUR 1800 million to EUR 900 million. This means that the money duration of the bond futures must be EUR 900 million.

The required size of the futures portfolio can be calculated using the money duration equation:

\[ \text{EUR 900 million} = \text{market value bond futures} \times 9 \]

The trader must, therefore, sell futures contracts with a total market value of EUR 100 million.

EXAMPLE

If the price of a bond is 98.70 and the bond has a duration of 4.6, the basis point value of this bond is:

\[ \text{BPV} = 98.7 \times 4.6 \times 0.0001 = 0.045. \]
This means that the price of the bond will decrease from 98.70 to 98.655 when the interest rate rises by 1 basis point.

A disadvantage of the modified duration and the way in which the basis point value is used above, is that it assumes implicitly that all zero coupon rates move in the same direction; in other words that the yield curve moves in a parallel way. The effect of a parallel shift is shown in figure 5.6.

Figure 5.6 shows that the value of the above bond as a result of a parallel interest rate rise of 1 basis point has fallen by EUR 47,417.60. This is also the basis point value for this bond.
In practice however, instead of assuming a parallel interest rate shift, sensitivity analyses to interest rate movements are made per time interval or bucket. Thus, separate analyses are made of the impact of a change in the one year rate, in the two year zero coupon rate, etc. By doing this, it will become clear that the interest rate sensitivity is almost always different in all time buckets:

<table>
<thead>
<tr>
<th>BUCKET (YEAR)</th>
<th>BASIS POINT VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 - 1.5</td>
<td>EUR 558.43</td>
</tr>
<tr>
<td>1.5 - 2.5</td>
<td>EUR 1,075.93</td>
</tr>
<tr>
<td>2.5 - 3.5</td>
<td>EUR 1,553.23</td>
</tr>
<tr>
<td>3.5 - 4.5</td>
<td>EUR 1,991.22</td>
</tr>
<tr>
<td>4.5 - 5.5</td>
<td>EUR 2,238.78</td>
</tr>
</tbody>
</table>

As might be expected, the table shows that the interest rate sensitivity of the bond principally lies in the five year bucket. After all, this is where the largest cash flow appears.

The above table is often referred to as a gap report. Financial institutions use these kinds of gap reports in order to determine how their interest rate exposure is spread across the various maturity periods. If a bank has a clear idea about the interest rate movement in a specific part of the yield curve, it can use this detailed information to fine tune its hedge transactions.

In addition to the basic point value that presents the change in value of a financial instrument or future cash flow as a result of a change of 1 basis point in the zero coupon rate, there is a comparable indicator. This indicator shows the change in value of a financial instrument or a single cash flow as a result of a change of 1 basis point in the credit spread. This indicator is called the credit BPV or CV01, although some banks still use the term PV01 for this.

### 5.7.6 Modified duration of FRNs

The problem with the valuation and the determination of the modified duration of a floating rate note (FRN) is that, with the exception of the first interest payment, the future cash flows arising from the interest payments have not yet been determined. The cash flows for an FRN with a maturity period of five years based on six month EURIBOR and a first fixing of 5% (daycount convention 30/360) are shown in figure 5.7.